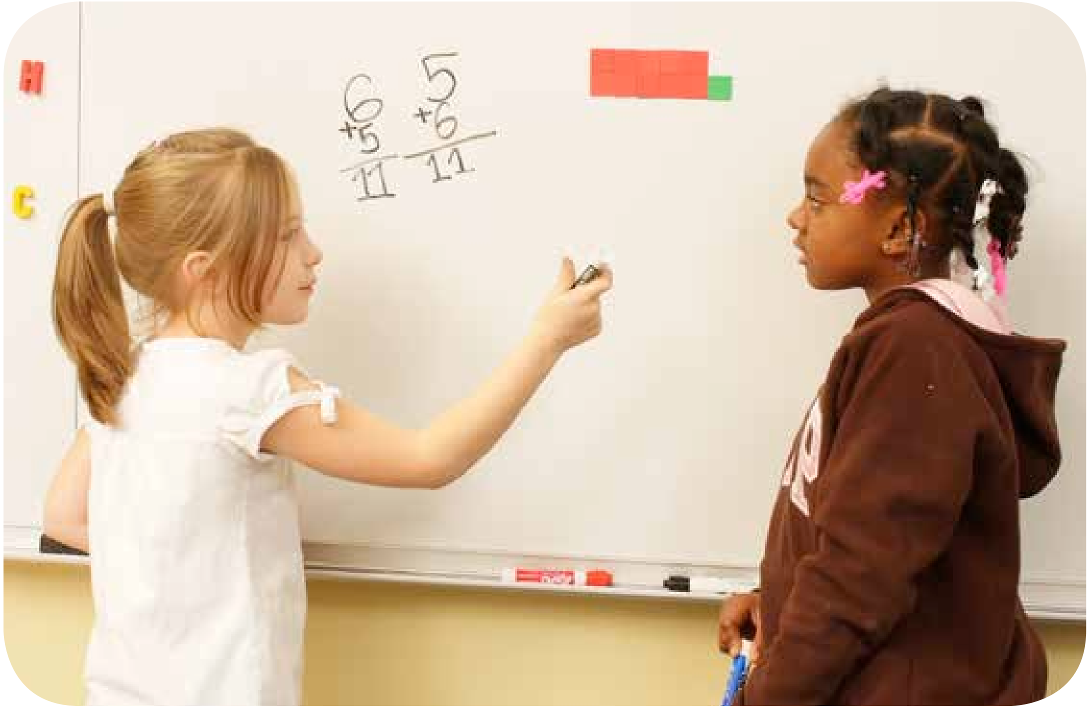


Developing Fluency with Basic Facts

Computational fluency, according to NCTM's Principles and Standards for School Mathematics, means more than quickly producing correct answers. It requires conceptual understanding and is exhibited through:

- Efficiency
- Accuracy
- Flexibility

As you guide your students to mathematical fluency, you will give them many opportunities to construct relationships among numbers to make sense of basic facts and be able to retrieve them. This article focuses on:

- Developing multiple aspects of fluency
- Building fluency through use of strategies, intuitive understanding, and mathematical models

Students who attain fluency with basic math facts have far fewer obstacles to building higher-order mathematical skills. Cognitive research indicates that math problem solving occurs in the working memory, which is able to process only a few calculations at a time. A student who cannot fluently grasp the first steps of a multistep problem may not be able to solve the problem simply because he or she isn't able to hold the required number of calculations in their working memory. We want students to know math facts much like they recognize sight words in reading—automatically, without thinking about them.

Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they do not know.

Children who struggle to commit basic facts to memory believe there are "hundreds" to be memorized because they have little or no understanding of the relationships among them. Children who commit the facts to memory easily are able to do so because they have constructed relationships among them and between addition and subtraction in general, and they use these relationships as shortcuts.

Fosnot & Dolk, 2001

Developing multiple aspects of fluency

Reflecting on the way NCTM characterizes fact fluency gives us a definition broader than the conventional "speed and accuracy." Efficiency implies that the student does not get bogged down in many steps or lose track of the logic of the strategy. An efficient strategy is one that your students can carry out easily, keeping track of subproblems and making use of intermediate results to solve the problem. Accuracy depends on several aspects of the problem-solving process, among them careful recording, the knowledge of basic number combinations and other important number relationships, and concern for double-checking results. Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Your students need to be flexible to be able to choose an appropriate strategy for the problem at hand and also to use one method to solve a problem and another method to double-check the results.

According to Catherine Twomey Fosnot and Maarten Dolk, memorization of basic facts usually refers to committing to memory the results of unrelated operations so that thinking is unnecessary. Isolated addition and subtraction problems are practiced one after another as if there were no relationships among them; the emphasis is on recalling the answers. Teaching facts for automaticity, in contrast, relies on thinking. Answers to facts must be automatic, produced in only a few seconds; counting is not sufficient. But reasoning about the relationships among facts is critical. A student who thinks of 9 + 6 as 10 + 5 produces the answer of 15 quickly, but thinking, not memorization, is at the core. The issue, according to these authors, is not whether facts should eventually be memorized but how this memorization is achieved: by drill, practice, and memorization, or by focusing on relationships.

The Bridges in Mathematics approach to fluency draws from the perspective that fact retrieval is far more likely to be successful when based on models and the use of strategies, as opposed to rote memorization and recall. Research by Constance Kamii as well as that of other educational experts lends support for focusing on number relationships to achieve fact fluency. Kamii compared two first grade classrooms in the same school. In one the teacher focused on number relationships and worked toward automaticity. In the other, students memorized facts with the help of drill sheets and flashcards. She found that students in the classroom in which automaticity was the goal significantly outperformed the traditionally taught students in being able to produce correct answers to basic addition facts within 3 seconds—76% met the goal in the first group compared with 55% in the second. Some of the most difficult facts for the traditional students were solved easily by the other group, with strategies like Doubles Plus or Minus One, working with structures of fives, and Making Tens.

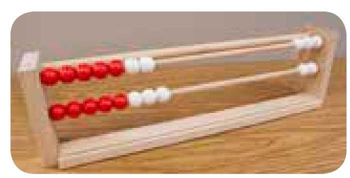
In the Bridges curriculum, students learn to think about numbers in terms of parts and missing parts. Such part-part-whole reasoning is not only fundamental to a rich understanding of numbers, but also fundamentally related to addition and subtraction as operations. Take, for example, the addition fact 8 + 5 = 13. Research has indicated that this combination is one that causes difficulty for many students. The Bridges teacher guides students to approach this troublesome combination from a range of perspectives, each of which is helpful in retrieving the correct answer. For example, after repeated use of the number rack, some of your students will see the number 8 as a combination of 5 and 3 more. Combining the two 5s, this number fact

becomes 5 + 5 + 3 = 13. Other students might use a compensation strategy to make this an easier problem to solve: take 2 from the 5, and add it to the 8. That leaves them with 10, and 3 more. It is this flexibility, built upon solid number sense and part-part-whole thinking, that allows students to reason their way toward mastery of the number facts.

This emphasis on facts and computation strategies to achieve fluency does not deny a role for memorization in the math curriculum. You'll find that, inevitably, some of your students will need to memorize facts for which they struggle to develop other strategies. As they learn to draw upon intuitive strategies and a strong sense of number, however, the number of facts they must memorize is reduced greatly. Your role is to help them see the basic facts not as something to be recalled, but as an expression of a mathematical relationship for which they have a conceptual anchor.

Using models, intuition, and strategies to develop fact fluency

The Bridges program extensively incorporates manipulatives and visual models such as the number line and the number rack to help students develop powerful reasoning strategies. They use these models as they break apart numbers, solve part-part-whole problems, and apply skip-counting strategies to model operations. Linking basic facts to representations on the models supports and feeds their intuitions about number relationships.



The number 15 as shown on the number rack.

The number rack, used in grades K–2, is a base ten model that allows students to create and actively manipulate numbers between 0 and 20. It combines features of the number line, counters, and base ten models. The number rack comprises 2 strings of 10 beads, each of which is strategically broken into a group of 5 red beads and a group of 5 white beads. These groupings invite students to think in sets of 5 and 10.

The number rack model is instrumental in helping students visualize number quantities, allowing them to "see inside" numbers—with an emphasis on 5s and 10s—and to do so quickly and efficiently. These skills are precursors to developing informal (and formal) strategies for multi-digit addition and subtraction. Repeated use of the model supports students to naturally acquire the foundation for these strategies that lead to fluency.

In the teaching materials for each grade level, you'll find further information on the number rack and other models in use and how they aid development of fact fluency: the models are described in the overall "Introducing Bridges in Mathematics" and "Number Corner Introductions" as well as in the introductory sections of Bridges units and Number Corner months. In addition, look for further Teaching & Learning articles that break out the models used at each grade level and trace connections among the models.

Strong conceptual understanding of the fact strategies (shown in the chart below) goes hand in hand with mathematical models to build fluency in basic facts. Students need lots of time and experience identifying and applying these strategies to eventually develop a conceptual understanding of number relationships.

When students repeatedly use visual representations on the number rack and build upon 10 as a foundation, the categories of addition facts listed below emerge.

FACT STRATEGY NAME	EXPLANATION	EXAMPLE
Add Zero facts	Adding 0 to any quantity does not change the value of the quantity.	7 + 0 = 7
Add Ten facts	Adding 10 to any quantity preserves the value in the ones column.	7 + 10 = 17
Add Nine facts	Adding 9 to any quantity is 1 less than adding 10.	7 + 9 = 16
Count On facts (+1, +2, +3)	Students can count on with meaning up to 3.	5 + 3 may be thought of as 5 … 6, 7, 8.
Make Ten facts	Combinations that make 10 are inherent in the number rack and readily visualized.	6 + 4 = 10
Doubles facts	The Doubles facts are visualized on the number rack as two equal rows of beads.	4 + 4 is seen as two groups of red beads on the number rack.
Doubles Plus or Minus One facts	Once the doubles are solidified, students compensate by adding or subtracting 1 to find the near doubles.	6 + 5 may be thought of as "1 more than 5 + 5" or "1 less than 6 + 6."
Leftover facts	After the previous seven strategies are well known, students realize that there are only a handful of facts remaining to be learned. These Leftover facts are often recalled by linking them to previously learned strategies.	To find the sum of 8 + 6, students can think about making 10 and then compensate, taking 2 from the 6, adding it to the 8, and then adding the remaining 4: 10 + 4 = 14.

When students learn the addition facts in the context of these strategies, they are not overwhelmed with the task of memorizing hundreds of different combinations. Instead, they can confidently use their mathematical reasoning and number sense to quickly find those sums that they cannot instantly recall.

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